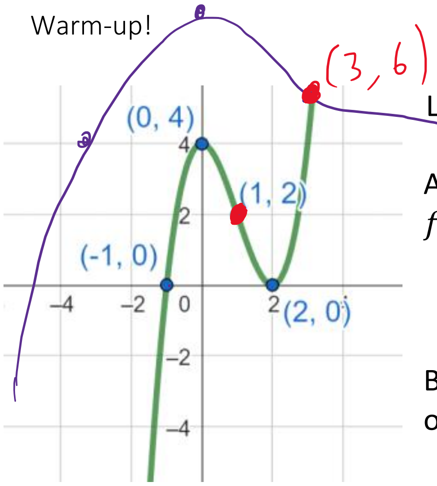
Warm-up!



Let $f(x)$ be the function graphed to the left.

A) Describe the end behavior of $f(-x)$ using limit notation.

$\lim_{x \to -\infty} f(-x) = \infty$

$\lim_{x \to \infty} f(-x) = -\infty$

B) Over what interval is the rate of change of $-f(x+2)$ positive and decreasing?

$(-1, 0)$

C) Find the coordinates of the point of inflection of the function f

$(1, 2)$ $f(x)$

$(3, 2)$ $f\left(\frac{x}{3}\right)$

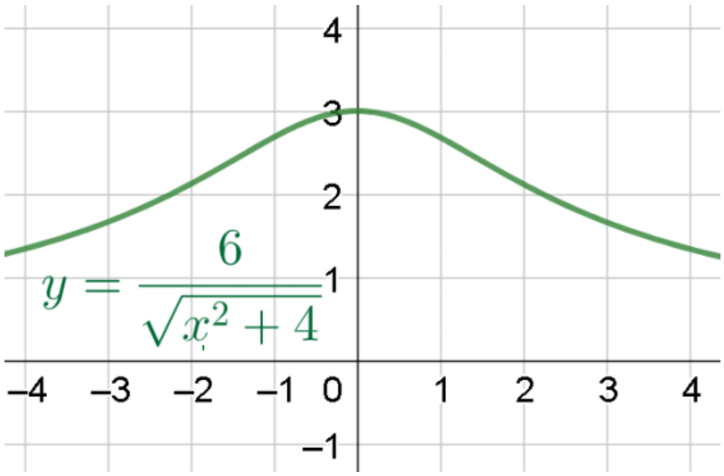
$(3, 6)$ $f\left(\frac{x}{3}\right) + 4$

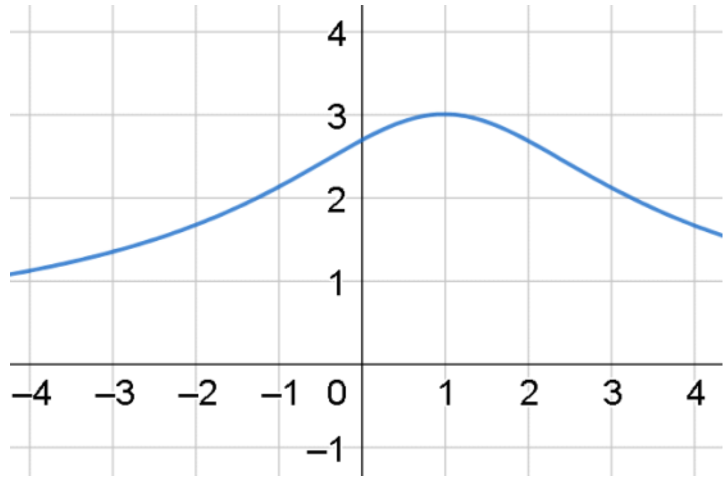
Periodic Graphs Day 2

- (3.1) Period, Frequency
- (1.12) More Transformations Review

Homework: Transformations and Intro to Periodic Graphs

Find the equation of the blue graph





A) $y = \frac{6}{\sqrt{(x-1)^2+4}}$

B) $y = \frac{6}{\sqrt{(x+1)^2+4}}$

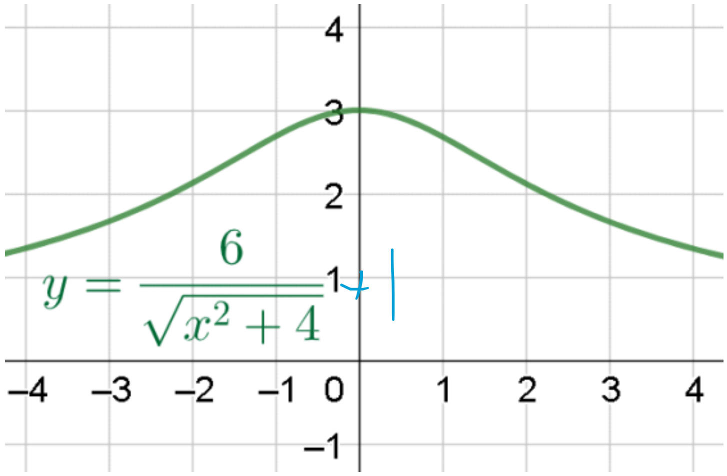
C) $y = \frac{6}{\sqrt{x^2+3}}$

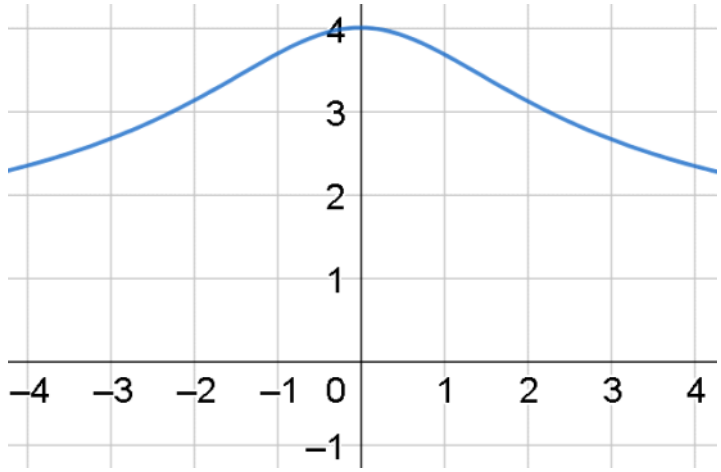
D) $y = \frac{6}{\sqrt{x^2+4}} + 1$

E) $y = \frac{6}{\sqrt{x^2+1}} - 1$

F) $y = \frac{6}{\sqrt{x^2+5}}$

Find the equation of the blue graph





A) $y = \frac{6}{\sqrt{(x-1)^2+4}}$

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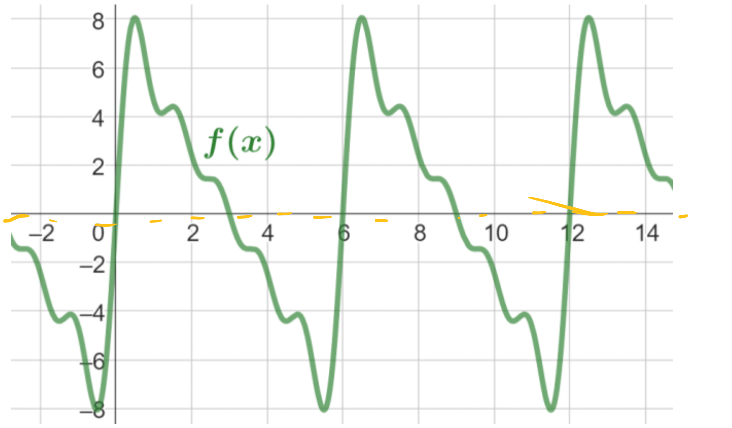
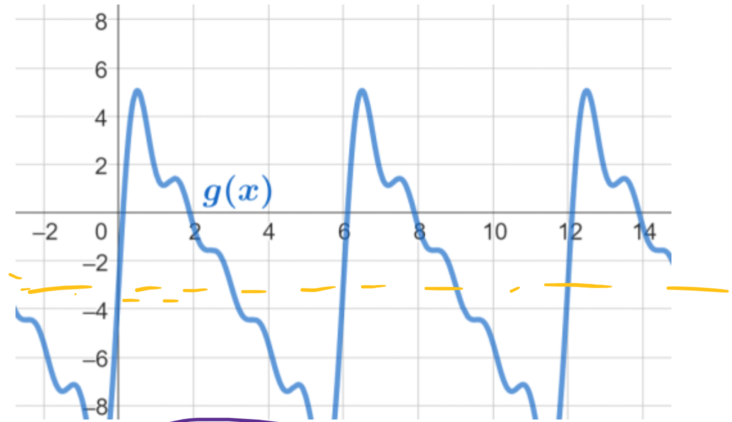
Find the equation of the blue graph in terms of $f(x)$

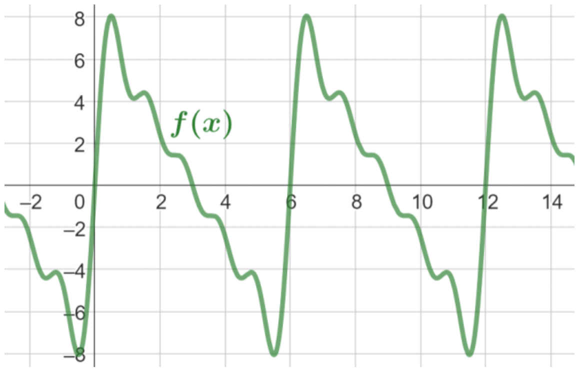


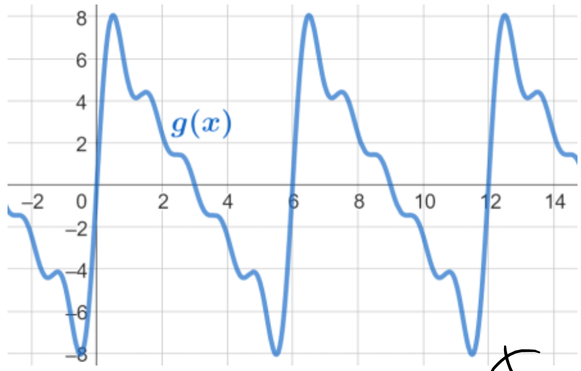


A) $y = f(x-3)$ B) $y = f(x+6)$ C) $y = f(x) - 3$

D) $y = f(x) + 3$ E) $y = f(x-6)$ F) $y = f(x-12)$

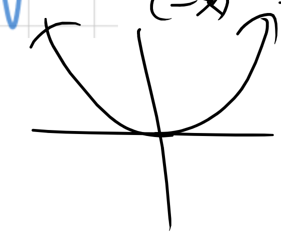
Find the equation of the blue graph in terms of $f(x)$





x^2

$(-x)^2 = x^2$



A) $y = f(x-3)$ B) $y = f(x+6)$ C) $y = f(x) - 3$

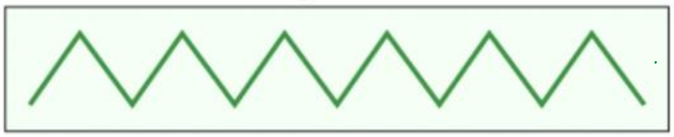
D) $y = f(x) + 3$ E) $y = f(x-6)$ F) $y = f(x-12)$

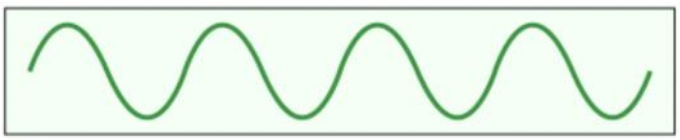
Periodic Functions

A function f is periodic when there exists a positive real number c such that $f(x+c) = f(x)$

Simpler: A function is periodic when it still looks identical after a horizontal translation

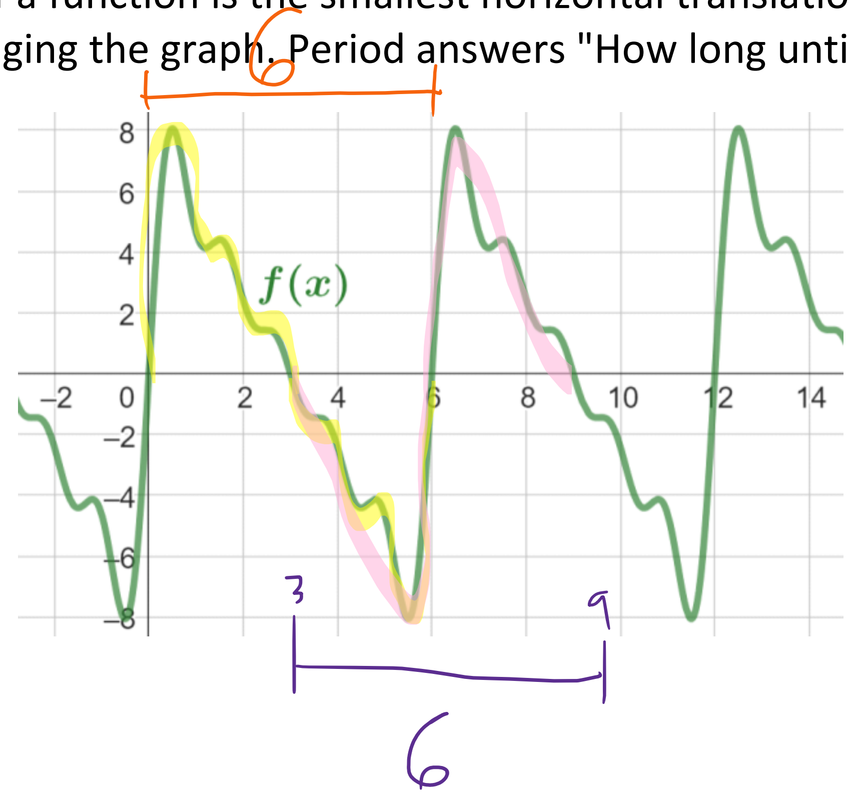
Even Simpler: A function is periodic when it is a repeating function



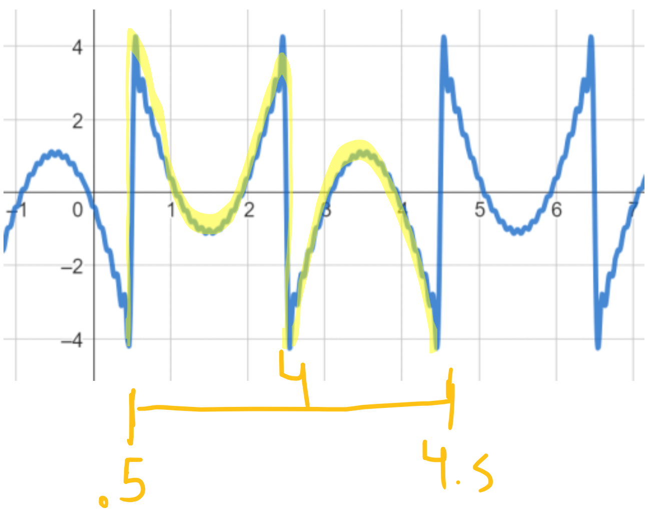


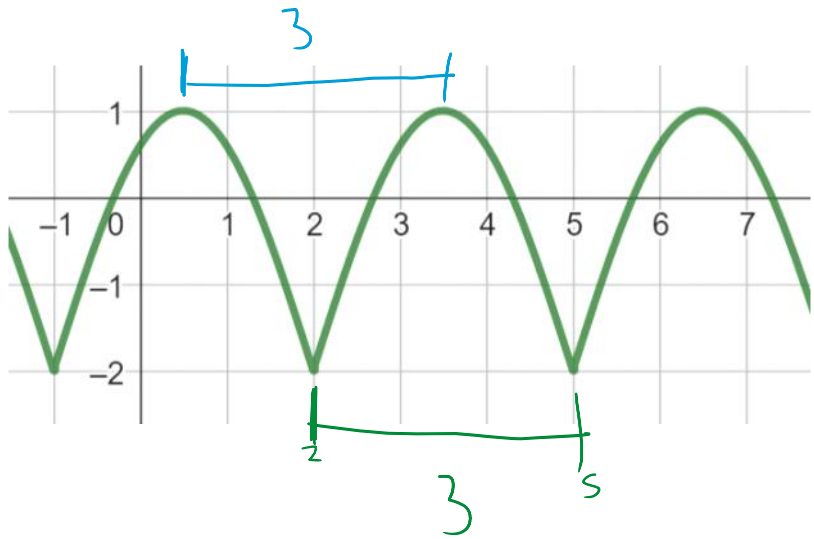
Period

The period of a function is the smallest horizontal translation that can be made without changing the graph. Period answers "How long until the graph repeats?"

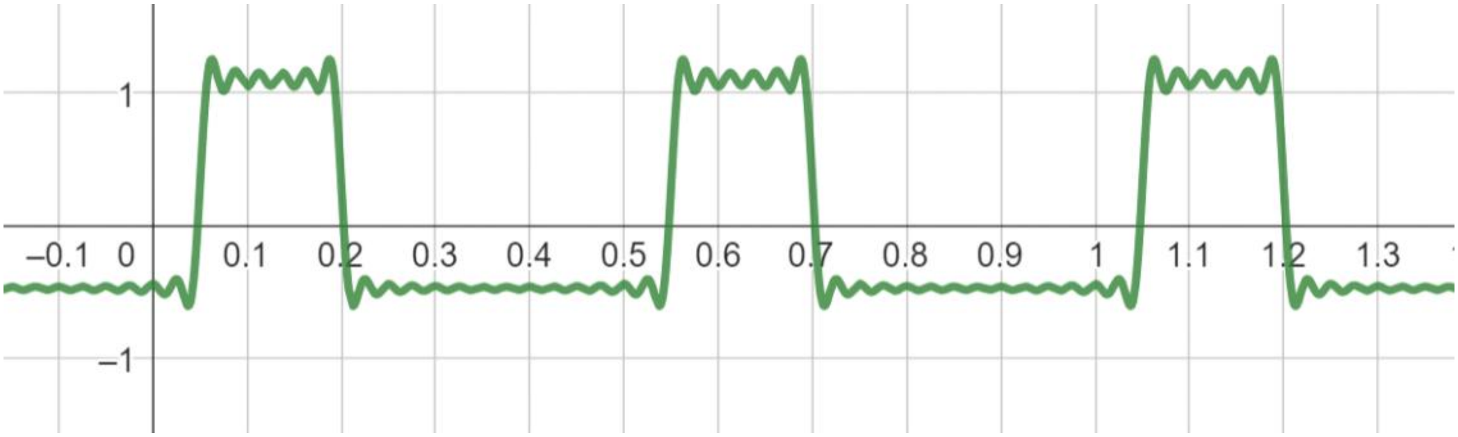


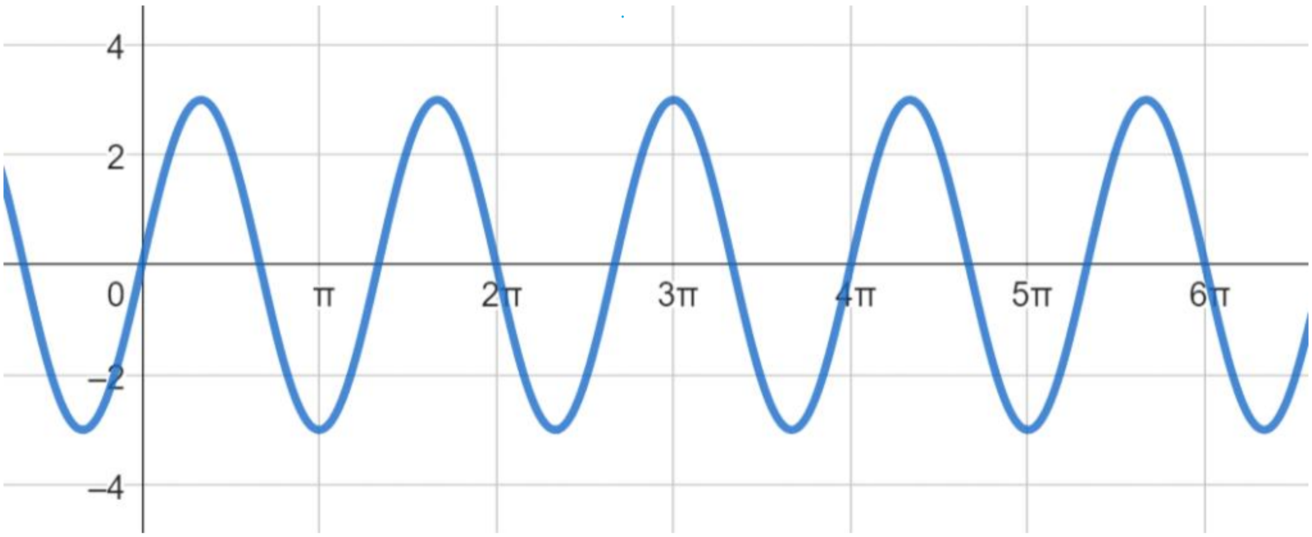
Find the period of each graph





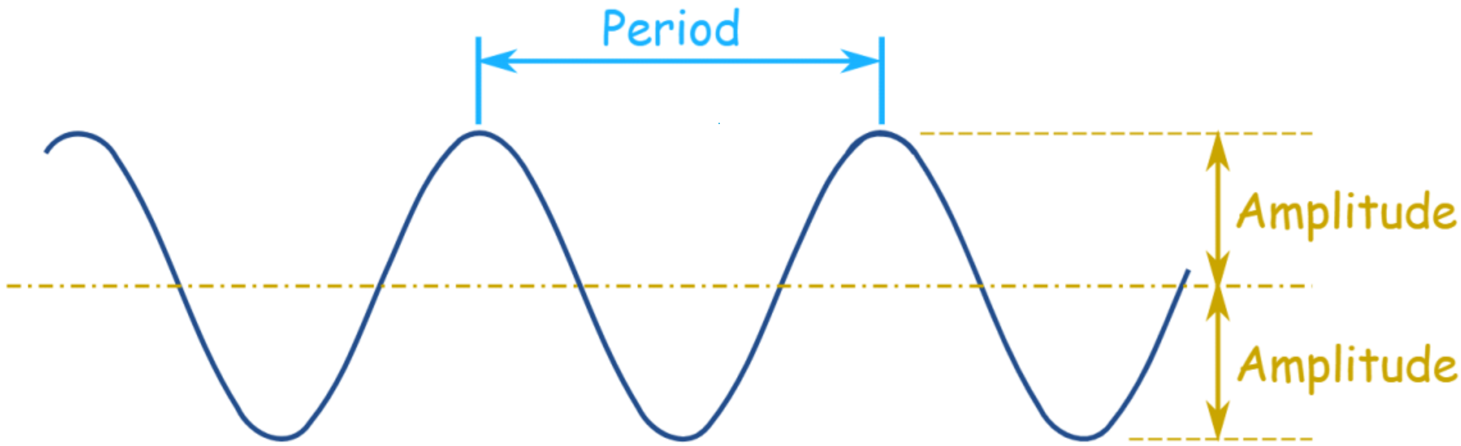
Find the period of each graph



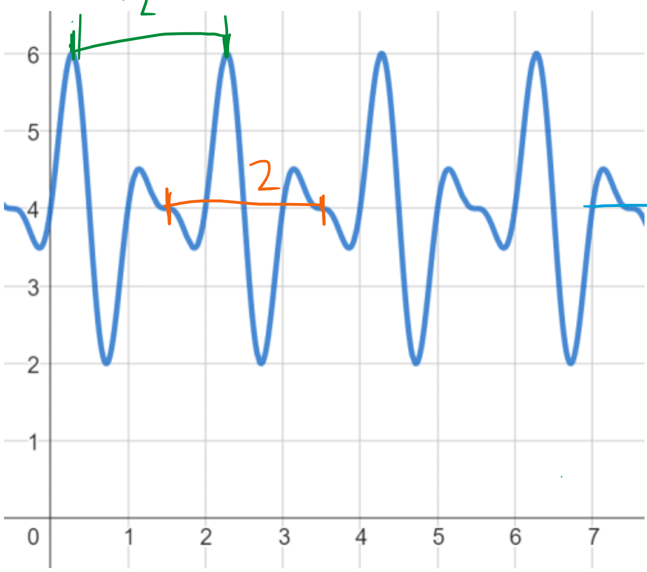


Amplitude

Amplitude: Half the distance between the absolute maximum and absolute minimum of a periodic function.



Below is the graph of $f(x)$



6

range [2,6]

What is the period of $f(x)$?

2   2

4

What is the amplitude of $f(x)$?

2   2

What is the period of $g(x) = 2f\left(\frac{x}{3}\right)$?

2

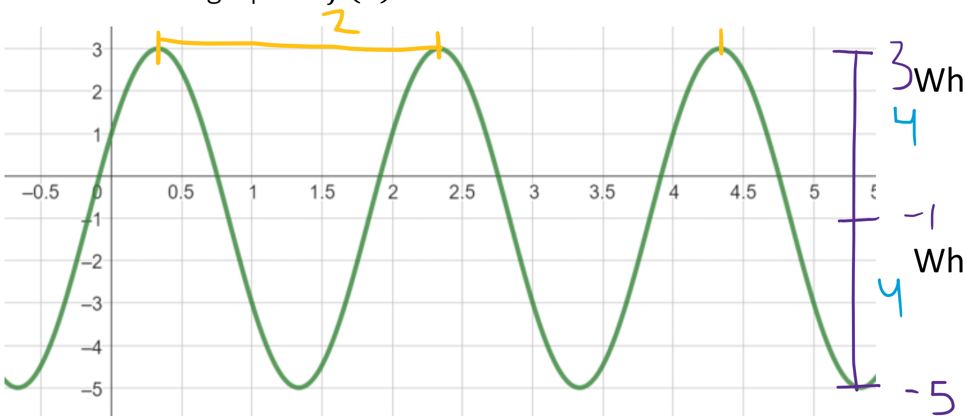
6

What is the amplitude of $g(x) = 2f\left(\frac{x}{3}\right)$?

range [4,12]

4

Below is the graph of $f(x)$



What is the period of $f(x)$?

2

What is the amplitude of $f(x)$?

4

What are the period and amplitude of $5f(3x)$?

Period: $\frac{2}{3}$

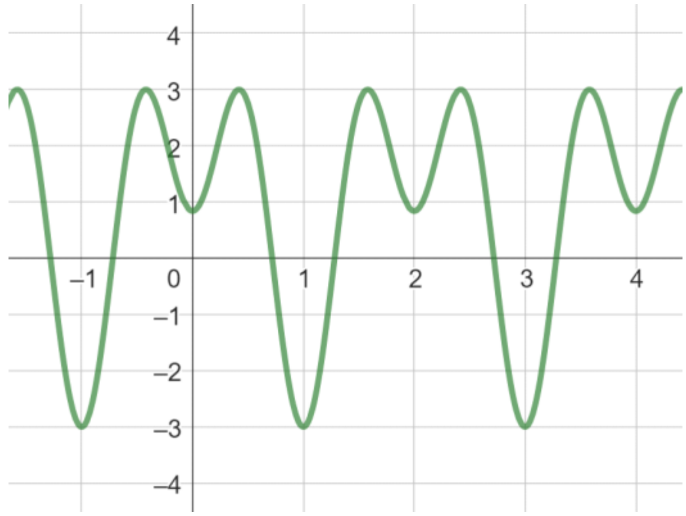
Amplitude: 20

What are the period and amplitude of $\frac{1}{2}f(2x)$?

Period: 1

Amplitude: 2

In green is the graph of $f(x)$. Find the period and amplitude of each function on the left.



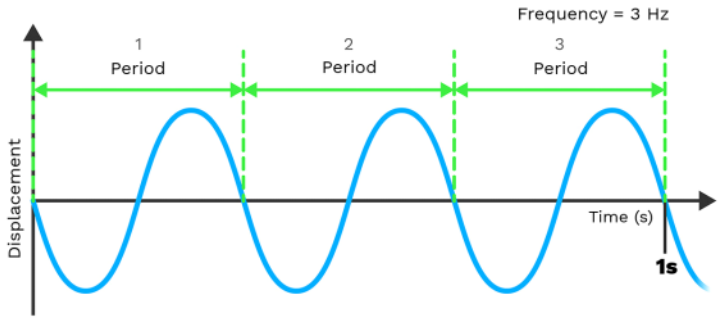
$f\left(\frac{x}{3}\right)$

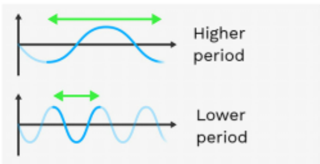
$4f(x-1)$

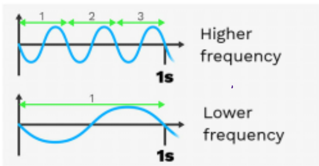
$3f(-x)$

$f(2x+2)$

Period and Frequency







Period and frequency are inversely proportional!

Period: How long it takes to complete one cycle

- A slow Ferris wheel might take a half hour to complete one full turn
- Every 24 hours, the earth spins once on its axis
- How cool loose fitting pants are repeats every quarter century

Frequency: How many cycles happen per unit of input (e.g. "per second" or "per year")

- A slow Ferris wheel might spin 2 times per hour
- The earth spins on its axis 7 times per week
- Loose pants are cool 4 times per century

Period and frequency practice

If the period of $f(x)$ is 3 months, what is the frequency of $f(x)$ measured in cycles per year?

$\frac{3}{1} \frac{\text{month}}{\text{cycle}} \rightarrow \frac{1}{3} \frac{\text{cycle}}{\text{month}} = \frac{4}{12} \frac{\text{cycles}}{\text{month}} \rightarrow 4 \frac{\text{cycles}}{\text{year}}$

If the frequency of $f(x)$ is $14 \frac{\text{cycles}}{\text{week}}$, what is the period of $f(x)$?

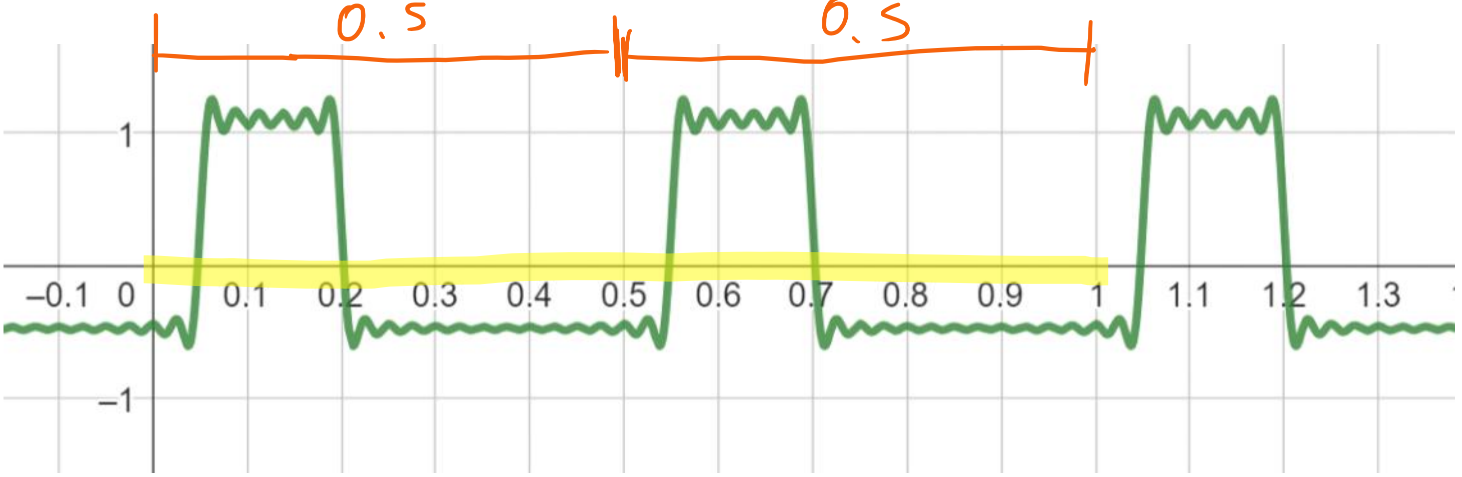
$\frac{14}{1} \frac{\text{cycles}}{\text{week}} \rightarrow \frac{1}{14} \frac{\text{week}}{\text{cycles}}$ Period = 12 hours = 0.5 days

If $f(x)$ repeats 12 times in 5 minutes, what is the period of $f(x)$, measured in minutes? What is the frequency of $f(x)$ measured in cycles per minute?

Freq = $\frac{12}{5} \frac{\text{cycles}}{\text{min}}$

Period = $\frac{5}{12} \frac{\text{min}}{\text{cycles}}$

Om sticks the probes of an oscilloscope into the brain of his roomba (don't try this at home) to measure the voltage. Below is the time versus voltage graph that reflects his measurements.



What are the period and frequency of this graph?

Period = 0.5 seconds

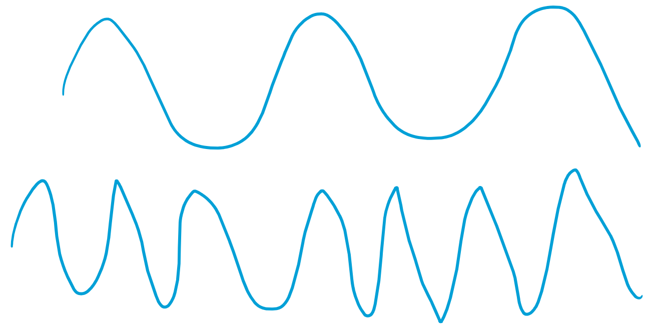
Frequency = 2 $\frac{\text{cycles}}{\text{sec}}$

Let $f(x)$ be a function with frequency 24 Hz

Find the frequency of $3f(x)$ = 24 Hz

Find the frequency of $f(3x)$ = 72 Hz

Find the frequency of $f\left(\frac{x}{3}\right)$ = 8 Hz



$f(x)$ is a function with amplitude 6 ft, period 0.5 days and frequency $2\,\frac{\text{cycles}}{\text{day}}$

Find the amplitude, period, and frequency of…

$5f(x)$ Amp = 30 ft P = 0.5 day f = 2 $\frac{\text{cycles}}{\text{day}}$

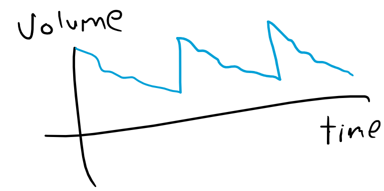
$f\left(\frac{x}{2}\right)$ Amp = 6 ft P = 1 day f = 1 $\frac{\text{cycle}}{\text{day}}$

$\frac{1}{3}f(x)$ Amp = 2 P = 0.5 f = 2

Which of the following scenarios would most likely be a periodic function? For that scenario, guess the period, amplitude, and frequency (rough guess, I don't expect you to magically know these things)

A) Modelling the weight of an elephant over time if the lifespan of an African elephant is 48 years and adult elephants reach 7 tons in weight.

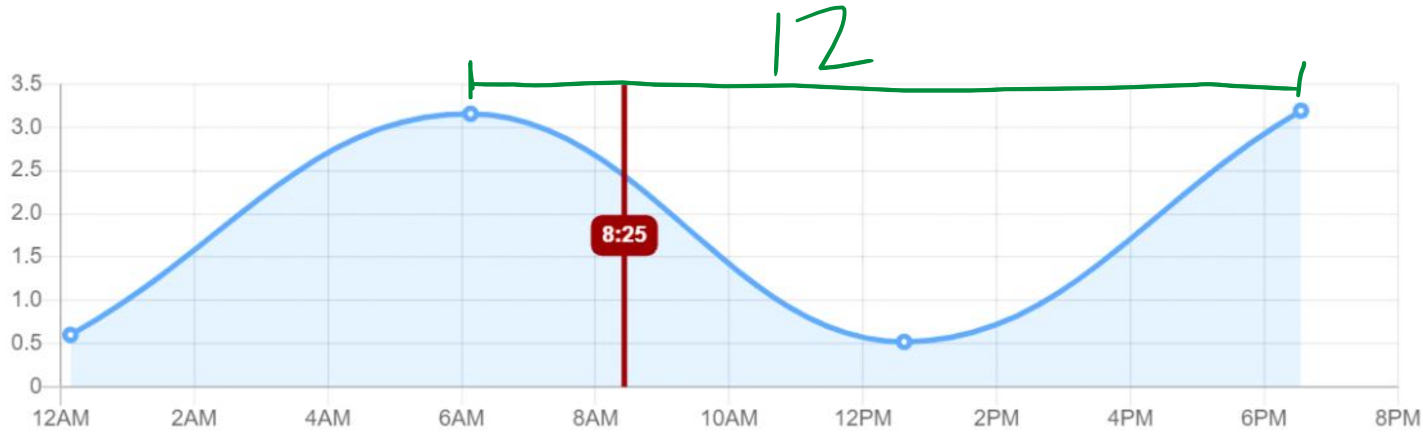
B) Modelling the volume of gasoline in your car over the time we have been in school so far this year.



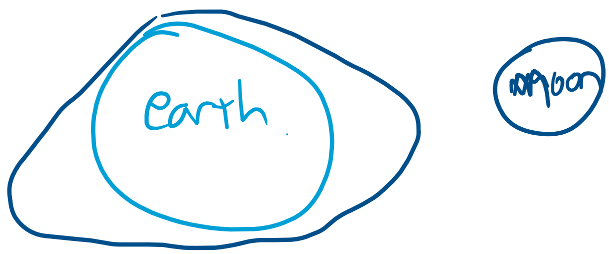
C) Modelling the level of plutonium-238 isotopes in spent nuclear fuel rods if the half-life of plutonium-238 is 87 years

D) Modelling the intensity of the sound coming from striking a gong over time if the gong has a diameter of 80 inches

Below is a graph of the tide height on Fort Lauderdale Beach. Find the period, amplitude, and frequency of the tide



https://oceanservice.noaa.gov/education/tutorial_tides/tides05_lunarday.html.



Which of the following scenarios would most likely be a periodic function? For that scenario, guess the period, amplitude, and frequency (rough guess, I don't expect you to magically know these things)

A) Modelling the height of a tennis ball over time after it is dropped from a height of 10 feet

B) Modelling the total value of a bank account over time if the account began with $\$200$ and accrues 2.5% interest each year

C) Modelling the volume of air in Travis's lungs as he sits at a concert if he is $6'5"$ and is in good cardiovascular health

D) Modelling the temperature of a cup of coffee over time after it begins at $75°$ C and is sitting in a room that is $22°$ C